

# APPLYING MARKOV CHAIN MONTE CARLO TECHNIQUES TO ESTIMATE INTERNAL CONSISTENCY RELIABILITY IN PHYSICS ACADEMIC SELF-CONCEPT INVENTORIES

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## ABSTRACT

The use of Cronbach's alpha ( $\alpha$ ) to estimate the overall internal consistency (OIC) of polytomously scored (PS) items with hierarchical structure has been a common practice among researchers for decades, despite advances in algorithm and software technologies. The use of coefficient alpha is hinged on the assumption that the indicators of an inventory should have equal factor loadings, uncorrelated measurement errors and that the items should measure only one latent construct. The assumptions appear to be violated by many researchers in practice. To avert apparent danger of either underestimation or overestimation of OIC reliability using  $\alpha$  in a scientific study for an inventory that contains more than one latent construct, McDonald's (1999) omega-hierarchical coefficient was adopted. Data collected were analyzed with eigen values, regression weights, error variances, polygon and trace plots available in factor analysis. The result indicated that ten items of Physics self-concept inventory showed approximate OIC reliability of .75 based on the Markov Chain Monte Carlo sample size of 100,000. Physics teachers should utilize the inventory to assess their students' academic self-concept. Researchers should also adopt the procedure to measure the OIC reliability of PS inventory with hierarchical structure.

**KEYWORDS:** MCMC, internal-consistency-reliability, Physics and academic self-concept.

## 1. INTRODUCTION

Physics academic self-concept (PASC) is a belief system that a learner holds which propels the learner to develop academic confidence in himself or herself as well as the ability to make effort towards achieving a Physics goal. PASC of the students is an important phenomenon within the context of classroom instruction and as such, it is related to Physics learning. Chun-Yen and Pei-Ling (2017) indicated that academic self concept was positively correlated to science achievement with large effect size. Thus, Physics, being a subset of Science is also influenced by student' academic self concept.

The method of estimating the internal consistency reliability of inventories including students' academic self concept inventory has partly been limited by small sample size and dimensionality of the inventory (Wahyu & Hamdollah, 2014). It is pertinent to describe the internal consistency reliability of an inventory, before delving into sample size and inventory dimensionality. Revelle (1997) defined the internal consistency of a test as the extent to which all of the items of a test measure the same construct, which is the general factor saturation. This implies that polytomous test items that are internally consistent, should have all their observed indicators load positively unto the second-order general construct. Nworgu (2015) defined internal consistency of a test from a reliability angle as the degree of consistency with which the items of an instrument measures a given trait. The trait of a polytomously scored test is conceptualized as a latent variable basically defined by few out of many unknown observable indicator-variables. The relationship between a single trait and its indicators corresponds to what is termed first-order structural equation model. However, there is a situation, where in a single inventory there are many related traits within it. At times, the traits may be hierarchical, leading to a general or common latent variable having other latent variables with their corresponding indicator variables as its indicators. The

arrangement of traits in the order just mentioned with its latent indicators is termed second-order or hierarchical structural equation model and they were used interchangeably. The formula for the internal consistency reliability estimation of the items of an inventory that has a single latent trait should be different from an inventory that has many traits and in most circumstances, hierarchical traits (Wahyu & Hamdollah, 2014).

The practical method of estimating the internal consistency reliability of polytomously scored items including likert-type, rating scales or inventories in the fields of Education and Social Sciences has been Cronbach's alpha. The original developer of alpha coefficient was Cronbach (1951). The underlying assumption for use of alpha coefficient included unidimensionality of the indicators, essentially tau-equivalence of indicators and that the measurement errors are uncorrelated (Zinbarg, Yovel, Revelle & McDonald, 2006). Unidimensionality assumption implies that the indicators of such a scale to be analyzed using coefficient alpha should measure only one latent construct. Any attempt to analyze a scale with more than one latent variable at a time violates its

use. Furthermore, tau-equivalence of indicators has two conceptual meanings. Zinberg, Revelle Yovel and Li (2005) described it as equality of factor loadings. It implies that the factor loadings of the indicators of a construct should be equal before Cronbach's alpha is used to test internal consistency reliability. However, to Schumacker and Lomax, (2010) the indicators that define a unit latent variable are declared tau-equivalent should they have similar individual item mean which can be represented by the term "intercept". In a situation where the indicators of one latent construct are not tau-equivalent, the use of alpha for internal consistency measure of such a scale without normalizing the data or constraining the mean is also faulty and will produce biased result. The last but not the least, is that the error terms should not be correlated. Correlating the error terms is one out of many procedures to make a bad model fit the data. It changes the structure of the measurement model by introducing additional "cause" variables in the measurement model. So, a model can have unifactorial structure, tau-equivalent indicators, but if it contains any correlated error terms, coefficient alpha will produce biased result as well. Quoting Cronbach (1951), "tests divisible into distinct subtests should be so divided before using alpha coefficient." Some researchers may argue that an inventory that has a hierarchical structure still has one latent construct that define other latent indicators. Such researchers should be guided by the fact that the number of latent variables in a hierarchically structured inventory is not unity. Therefore, unidimensional assumption in a hierarchically structured inventory is violated if alpha is used for internal consistency measure. The unidimensionality assumption of Cronbach's alpha implies that the indicators of a single latent or unobservable construct should be homogenous. So, the application of coefficient alpha should not go beyond a homogenous and single latent variable defining its indicators. Common observation of researchers' use of Cronbach's alpha in Education field to test internal consistency reliability of items which are not dichotomously scored indicates that its use in determining the internal consistency reliability of sub-constructs of an inventory is appropriate. What is perhaps scientifically unacceptable is the use of Cronbach's alpha to determine the internal consistency reliability for the overall sub-constructs of an inventory because it violates its usage assumption. To further buttress the idea that most researchers misuse Cronbach's alpha, Schmitt (1996) noted that alpha was practically employed to estimate internal consistency of a test in an arbitrary way: even when the unidimensionality assumption is violated. Alpha appears to be misused by many researchers because they appear not to conduct exploratory factor analysis to determine the dimensionality of their inventories prior to its use. It is the result of the exploratory factor analysis (EFA) that will scientifically inform the researcher dealing with inventories or scales if the items are loaded on one latent construct or not. When coefficient alpha is used in a multidimensional manner, it will generally underestimate the true value of internal consistency reliability of an inventory (Gerbing & Anderson, 1988). Alpha also overestimates the proportion of variance for a general factor when the indicators are multidimensional (Chronbach, 1951; Revelle, 1979). So, the use of coefficient alpha in a multiple dimensional context produces biased result. The dimensionality of a scale determined at the initial level of the EFA corresponds to first-order dimensionality. Armor (1974) suggested that researchers who intended to use Cronbach's alpha should first run exploratory factor analysis to determine its dimensionality. Moreover, there is the need to confirm the result of EFA through the use of confirmatory factor analysis (CFA). It is at the level of CFA that a second-order latent variable termed general factor (g) can be added to produce an effect on each latent variable or the indicators extracted using EFA.

McDonald's (1999) omega hierarchical coefficient ( $\omega_h$ ) came into being to address what seems to be a stringent condition for using coefficient alpha. McDonald's omega hierarchical coefficient takes into cognizance the

complex nature of inventories for reliability estimation. It is computed using the factor loadings in a CFA model (Brunner & Süß, 2005). The CFA model is usually a well fitted second-order CFA model. The squared sum of factor loadings of g on the observed variables divided by the squared sum of factor loadings of g on the observed variables plus squared sum of factor loadings of each of the extracted latent variables on the observed variables plus the sum of the error or uniqueness variances on the observed variables represents  $\omega_h$ . The above explanation on how to compute omega hierarchical coefficient was deducted from an earlier equation used by Wahyu and Hamdollah (2014) for measuring the internal consistency reliability of items with multiple dimensions. The omega hierarchical equation is expressed as follows

$$\omega_h = (\sum_{i=1}^p \lambda_{gi})^2 / \sum_{j=1}^k (\sum_{i=1}^p \lambda_{gij})^2 + \sum_{i=1}^p e_i$$

Where  $\lambda_g$  is factor loading of g-indicators on j-factor,  $\lambda_i$  is factor loading of i-indicators on j-factor and e is the uniqueness or error variance. Thus omega hierarchical coefficient is CFA-ladden internal consistency reliability. In a bid to fill the gap of assessing the internal consistency reliability of an inventory whose EFA result showed the presence of more than one latent construct, MacDonald (1999) indicated that omega-hierarchical coefficient ( $\omega_h$ ) was an alternative to alpha. However, earlier studies on the estimation of  $\omega_h$  including Wahyu and Hamdollah (2014), and Watkins (2017) utilized maximum likelihood estimations. The drawbacks occasioned by the use of maximum likelihood estimation are in terms of sampling and the representativeness of the population

parameters in addition to higher standard error of parameter estimation relative to Bayesian (Nnadi & Anamezie, 2018). Even when there is no sampling and maximum likelihood estimation is used in data analysis, the standard error of parameter estimates generated usually does not make the estimates to be very close to the unknown true population parameter. Bayesian estimation utilizes Markov Chain Monte Carlo (MCMC) sampling, specifically Gibb's sampler to generate MCMC sample and the posterior values of parameters. MCMC sample is an artificial sample size generated to match the cycles and values of estimation of posterior values of parameters. The trace graph of each parameter estimate provides visual information on the size of the MCMC sample used in parameter estimation. The size of the MCMC sample corresponds to the iteration level. During the estimation/sampling cycle, normally distributed prior was selected for every parameter to correspond with the normally distributed posterior output to avoid computation complexities. The MCMC sample kept on increasing during an estimation process until the posterior parameter values appeared stable. Then, the sampling process was halted against the MCMC sample. The MCMC sample is usually larger than the population and can represent it. The fit of the model to the data is very important before accepting the parameter estimates. From a Bayesian point of view, the posterior predictive distribution plot with the mean value of the estimate coinciding with the peak of the distribution provides a visual evidence of a good fit of the model to the data (Lynch, 2007). Based on the foregoing, the problem of the study was to use MCMC approach to estimate McDonald's omega-hierarchical coefficient for internal consistency reliability measurement of Physics academic self-concept inventory.

## 2. PURPOSE OF THE STUDY

The study sought to: (i) develop and test-run hierarchical confirmatory factor analysis model of Physics academic self-concept inventory using Bayesian estimation. (ii) evaluate the goodness of fit of the CFA model. (iii) determine the MCMC sample at the point of convergence of the model (iv) estimate McDonald's omega-hierarchical coefficient from the CFA model.

## 3. RESEARCH QUESTIONS

Four research questions guided the study. They included: (i). What is the parsimonious hierarchical confirmatory factor analysis model of Physics academic self-concept inventory using Bayesian estimation? (ii) What is the goodness of fit of the CFA model? (iii) What is the MCMC sample at the point of convergence of the model? (iv) What is the point estimate of McDonald's omega-hierarchical coefficient from the CFA model?

## 4. MATERIALS AND METHODS

The study partly adopted an instrumentation and a fully Bayesian experimental designs. An instrumentation design seeks to develop and validate an instrument to be used to collect data. In a fully Bayesian experimental design, normal prior distribution of the parameters was merged with the data to determine the posterior values of the model's parameters via MCMC. The experimental nature of the design is described below. The first-order CFA model was manipulated by constraining the exogenous latent variables' mean and variance to '0' and

'1' respectively. This was done to allow all the factor loadings to freely vary. The mean of the exogenous error terms on the observed variables were set to '0'. The variances of the error terms were allowed to freely vary. Also, the regression weights of the error terms on each observed variable in the first-order CFA model was constrained to '1'. In addition to the first-order model constraints, second-order CFA model had the mean and variance of the general factor (g) constrained to '0' and '1' respectively. Since second-order CFA model could not run, through trial-and-error the data analyst constrained the regression weights of  $\lambda_8$  and  $\lambda_4$  in the model to a value of .1 and the model ran. The target population for the study was two thousand, three hundred and eighty-two senior secondary one students consisting of (950 males and 1432 female) in ten public secondary schools in Enugu East Local Government Area of Enugu state (Post primary schools management board, 2015). The sample for the study consisted of 86 SS1 students drawn using multi-stage during 2017/2018 academic session. In stage one, simple random sampling was used to sample Enugu education zone out of six education zones in Enugu state. Enugu education zone is made up of three local Government Areas. Stage two involved sampling Enugu East using simple random sampling. Stage three involved sampling three schools out of ten in Enugu East using purposive sampling. The reason for purposively sampling was for convenience of the researchers. SS1 students were used because all the students in the class offered Physics. The instrument used to collect data in this study was the original academic self-concept scale which was developed by Liu and Wang (2005). The scale had two sub-constructs: academic effort and academic confidence with ten items each measured on a 7 point continuum. Both items of the scale were mixed up with even numbers reflecting academic effort sub-scale and odd numbers belonged to academic confidence. Negatively worded items were built into the scale to avoid unengaged responses. The original instrument was adapted in this study. The adaptation involved changing the items on the scale to reflect Physics and changing the inventory's continuum from seven-point to four-point. The data collected from the students were first analyzed with eigen values using an exploratory factor analysis

approach in 'Psych' package in R computer program version 3.4.3 via R-studio version 1.0.153 to determine the number of possible sub-factors. The minimum residuals factor method and promax rotation were adopted. Two sub-factors with a total of twelve items were extracted based on eigen-values (variance of an eigen-vector) above .35. However, three factors were deleted on condition of having eigen-values above the value of 1.0 (Heywood case). Four items that loaded on two factors at a time were deleted. One item was also deleted on the condition of having very low positive eigen-value. The confirmatory factor analysis procedure followed the exploratory factor analysis to confirm the initial result. At the confirmatory level of data analysis, polygon and trace plots, regression weights and error variances were used. The software used was analysis of moment structures version 20. Two more factors in the hierarchical CFA model: 'I am interested in my Physics lesson' and 'I follow Physics lessons easily' were constrained to .1 in the CFA model before the model could run. Hence, the number of the items in the inventory further reduced to ten indicating that only ten items in the result of exploratory factor analysis were confirmed.

## 5. RESULTS

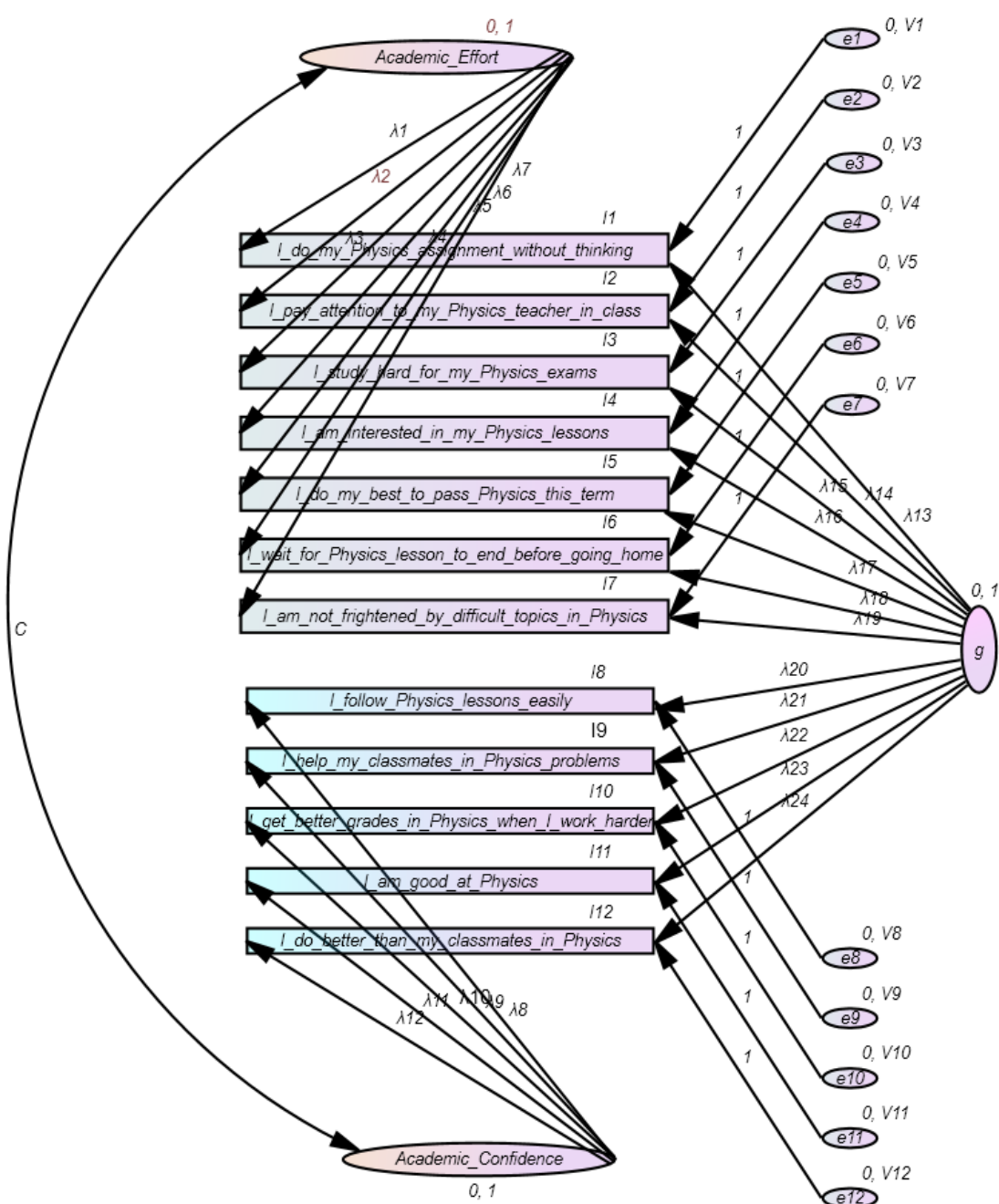
The results were presented according to the research questions that guided the study.

### Research question 1 (RQ1): What is the parsimonious hierarchical confirmatory factor analysis model of Physics academic self-concept inventory using Bayesian estimation?

The parsimonious hierarchical confirmatory factor analysis model of Physics academic self-concept inventory estimated using Bayesian estimation is shown in figure 1. Table 1 shows the estimated parameter values in the parsimonious model. The covariance between academic self-concept and academic confidence is .30. The values of the factor loadings of the items on academic effort ( $\lambda_1$ -  $\lambda_7$ ) ranged between .13 to .29 whereas the values of the factor loadings of the items on academic confidence ( $\lambda_9$ -  $\lambda_{12}$ ) ranged between .07 to .25. The range of values of the loadings of the general factor, g on the items ( $\lambda_{13}$ -  $\lambda_{24}$ ) was from .13 to .35. The item intercepts ranged between 1.52 to 2.67. Appendix A shows the full posterior values of the model's parameters. Table 1 was extracted from it.

*Table 1: Estimated parameter values in the parsimonious model*

| Factor loadings, $\lambda_s$ | Factor loadings, $\lambda_s$ | Error term variances, $V_s$ | Item Intercept (mean) $I_s$ | Covariance between sub-constructs, C |
|------------------------------|------------------------------|-----------------------------|-----------------------------|--------------------------------------|
| $\lambda_1=.17$              | $\lambda_{15}=.25$           | $V_1=.05$                   | $I_1=2.48$                  | C= .30                               |
| $\lambda_2=.25$              | $\lambda_{16}=.13$           | $V_2=.06$                   | $I_2=1.55$                  |                                      |
| $\lambda_3=.21$              | $\lambda_{17}=.22$           | $V_3=.02$                   | $I_3=1.56$                  |                                      |
| $\lambda_5=.19$              | $\lambda_{18}=.35$           | $V_4=.03$                   | $I_4=1.54$                  |                                      |
| $\lambda_6=.29$              | $\lambda_{19}=.16$           | $V_5=.05$                   | $I_5=1.59$                  |                                      |
| $\lambda_7=.13$              | $\lambda_{20}=.24$           | $V_6=.26$                   | $I_6=1.60$                  |                                      |
| $\lambda_9=.25$              | $\lambda_{21}=.54$           | $V_7=.01$                   | $I_7=1.52$                  |                                      |
| $\lambda_{10}=.07$           | $\lambda_{22}=.17$           | $V_8=.04$                   | $I_8=2.61$                  |                                      |
| $\lambda_{11}=.08$           | $\lambda_{23}=.18$           | $V_9=.02$                   | $I_9=2.41$                  |                                      |
| $\lambda_{12}=.14$           | $\lambda_{24}=.33$           | $V_{10}=.02$                | $I_{10}=2.51$               |                                      |
| $\lambda_{13}=.20$           |                              | $V_{11}=.01$                | $I_{11}=2.42$               |                                      |
| $\lambda_{14}=.29$           |                              | $V_{12}=.15$                | $I_{12}=2.67$               |                                      |



**Fig 1: Second-order structural model of Physics academic self-concept inventory.**

**Research question 2 (RQ2): What is the goodness of fit of the CFA model?**

The goodness of fit of the CFA model to data is visually shown in figure 2 using posterior predictive distribution referred to as polygon plots. The posterior predictive distribution peaks for  $\lambda_1$ ,  $\lambda_{10}$ ,  $\lambda_{13}$  and  $V_2$  approximately overlapped with their estimated values of .17, .07, .20, and .06 respectively.

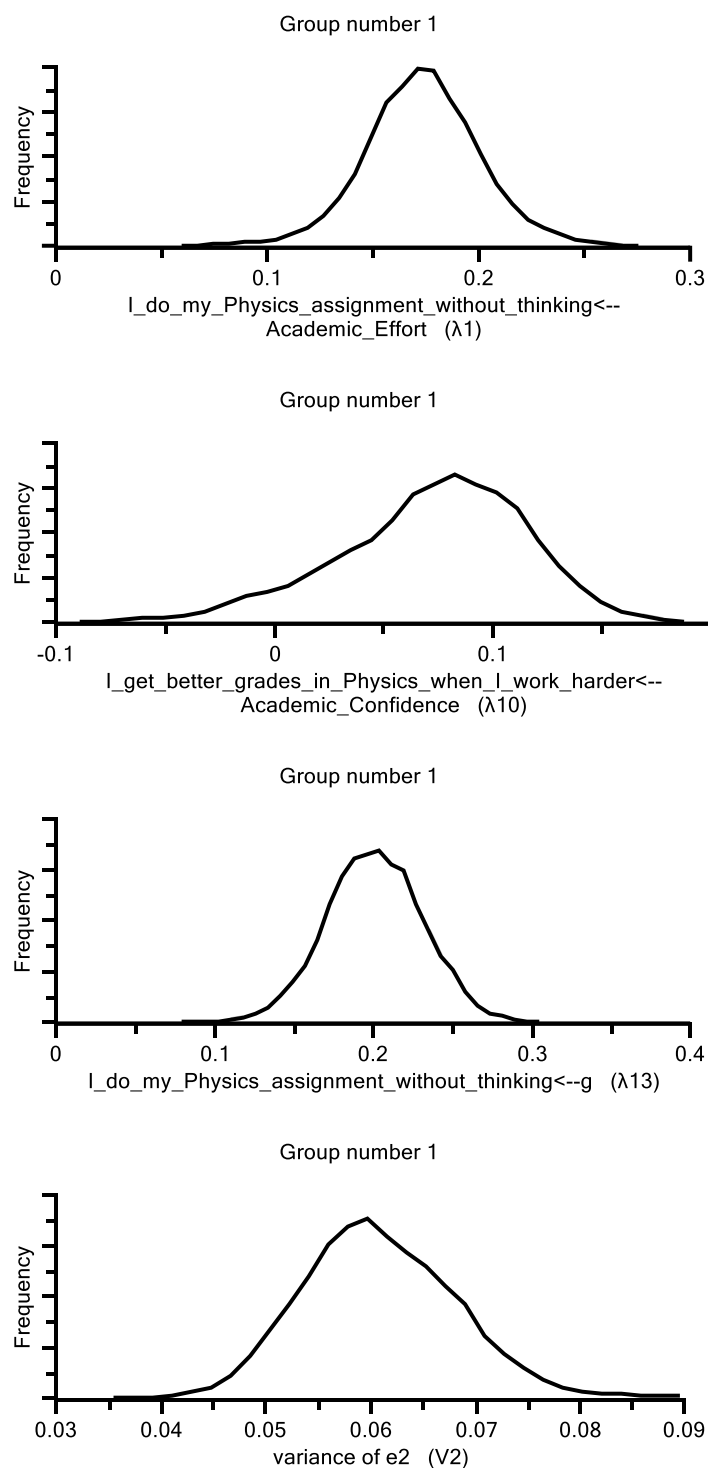
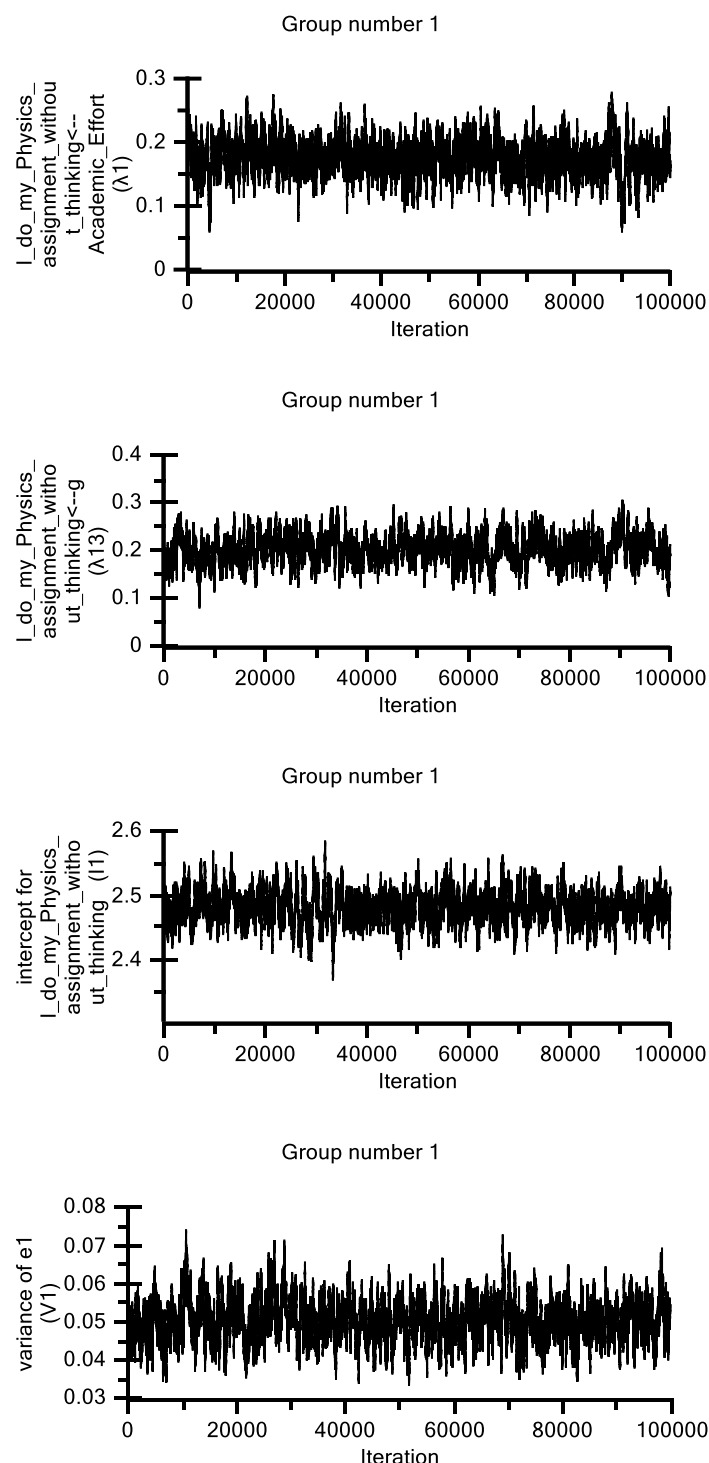


Figure 2: Posterior polygon plots of some estimates.

**Research question 3 (RQ3): At what MCMC sample did the model converge?**

Figure 3 is used to answer the research question 3. The iteration level for selected paths:  $\lambda_1$ ,  $\lambda_{13}$ ,  $I_1$  and  $V_1$  were 100000 each



*Figure 3: Trace plots of some estimates*

**Research question 4 (RQ4): What is the point estimate of McDonald's omega-hierarchical coefficient from the hierarchical CFA model?**

Table 2 is used to answer research question 4. From Table 2, the squared sum of the factor loadings of the general factor,  $g$  on  $\lambda_1, \lambda_2, \lambda_3, \lambda_5, \lambda_6, \lambda_7, \lambda_9, \lambda_{10}, \lambda_{11}$  and  $\lambda_{12}$  was 7.236. The squared sum of the factor loadings of academic effort,  $AE$  on  $\lambda_1, \lambda_2, \lambda_3, \lambda_5, \lambda_6$ , and  $\lambda_7$  was 1.538. Also the squared sum of the factor loadings of the academic confidence,  $AC$  on  $\lambda_9, \lambda_{10}, \lambda_{11}$  and  $\lambda_{12}$  was .292. The sum of error variance on  $\lambda_1, \lambda_2, \lambda_3, \lambda_5, \lambda_6, \lambda_7, \lambda_9, \lambda_{10}, \lambda_{11}$  and  $\lambda_{12}$  was .65. Therefore, the point estimate of McDonald's omega-hierarchical coefficient from the hierarchical CFA model was approximately equal to .75.



statistical method relative to traditional methods (maximum likelihood included) of model parameter estimation adopts MCMC approach. MCMC is more likely to reflect the true population estimates of reliability, since it uses artificial samples in its computation of parameter values. Lower standard error of estimation is achieved with Bayesian estimation. Majority of the estimated paths (Appendix A) in the model had almost zero (2 decimal places) standard error, SE with very few paths having .01 or .02 as their SEs. So the lower standard error of Bayesian estimation makes it robust both as a point and interval estimators. The mean values of the results of the cycles of path estimation all lie within the 95% lower and upper confidence intervals.

## 7. CONCLUSION

It is very difficult in practice to obtain essentially tau-equivalent factor loadings on a single construct, as one of the conditions for using Cronbach's alpha, hence the adoption of McDonald's omega hierarchical coefficient when the assumptions for the use of alpha fails. There is the need for Physics teachers to assess their students' self concept level with the self-concept inventory, since it is a covariate of Physics achievement. Researchers should adopt McDonald's omega hierarchical coefficient for testing the internal consistency reliability of an inventory with multiple dimensions. Editors of Scientific Journals should make sure that the test for the assumptions of using alpha is met by researchers interested in using inventories to collect data before alpha coefficient is used.

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