MHD Flow Characteristics of Nanofluids over Stretching Permeable Sheets under Chemical Reaction Influences

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ABSTRACT

In this work, we addressed the characteristics of two dimensional transient boundary layer flow of nanofluid over a non-linear stretching surface with magnetic field in presence of chemical reaction. By as set of similarity transformation, the proposed leading PDEs of flow phenomena are converted into non-linearly ODEs with boundary condition and then the solved numerically by using Keller Box method. The numerical solutions of the non-dimensional system equations have been illustrated. MATLAB software as well-recognized scheme is operated to solve problem for numerous values of governing parameters. The study reveals that the governing parameter, namely, Magnetic parameter, Wall mass transfer parameter, Prandtl number, Lewis number, Brownian motion parameter, Thermophoresis parameter and chemical reaction parameter, have major effects on the heat transfer and the nanoparticle volume fraction. For thermophoresis effect, the thermal boundary layer thickness becomes larger and the results are presented in both graphical and tabular form.

Keywords: *Lorentz force, Nanofluid, Heat Transfer, Keller-box method, Chemical Reaction.*

I. INTRODUCTION

Heat and mass transfer of the boundary layer flow over a stretching sheet has been generated an important research area due to its wide ranging applications in various engineering processes like polymer processing, extrusion of plastic sheets, metallurgical process, paper production. Such flows arise either due to unsteady motion of the boundary or the boundary temperature. Sakiadis [1] was the first person to study the boundary layer flow over a solid surface by taking velocity as constant. Crane [2] extended the work of Sakiadis by studying the boundary layer flow over a stretching sheet which moves in its plane varying linearly with velocity from a fixed point. Analytical and experimental results for the heat transfer flow in stretching sheet were examined by Tsou et al. [3] and mass transfer flow at the stretched sheet were studied by Erickson et al. [4]. Heat and mass transfer in hydrodynamic fluid flow over a stretching sheet with suction effect discussed by Gupta and Gupta [5].

The MHD parameter is one of the important factors by which the cooling rate can be controlled and the product of the desired quality can be achieved. The study of Magnetohydrodynamics MHD boundary layer flow on a continuous stretching sheet has attracted considerable attention during the last few decades due to its numerous applications in industrial manufacturing processes. Crane [6] was first to study the boundary layer flow caused by a stretching sheet which moves with a velocity varying linearly with the distance from a fixed point. Jafar et al. [7] studied the MHD flow and heat transfer over stretching/shrinking sheets with external magnetic field, viscous dissipation and Joule Effects. Prasad et al. [8] discussed the effect of variable viscosity on MHD viscoelastic fluid flow and heat transfer over a stretching sheet. Sharma and Singh [9] analyzed the effects of variable thermal conductivity and heat source / sink on MHD flow near a stagnation point on a linearly stretching sheet.

Nowadays, the study of convective transport of nanofluids plays an important role in cooling and heating applications. The convectional heat transfer fluids such as oil, water, and ethylene glycol mixture are having poor heat transfer fluids. This is because they have low thermal conductivity. In 1995, Choi [10] proposed a new type of fluid by adding nanoparticles into liquids and is termed as nanofluid. The enhancement of thermal conductivity of base fluids is found in [11 and 12]. Metals and metal oxides are commonly used in the preparation of nanofluid due to their outstanding properties such as thermal conductivity, electrical insulation and high cost performance ratio [13]. Khan and Pop [14] studied the nanofluid flow over a stretching sheet. Abdul Hamid et al [15] examined the stagnation point flow of a nanofluid over a permeable stretching/shrinking sheet. Zaimi et al. [16] `examined the boundary layer flow of a nanofluid over a stretching/shrinking sheet with suction/injection. They found that the suction parameter retards the fluid motion. Ferdows et al. [17] discussed the heat transfer characteristics of a magneto-nanofluid over a stretching sheet. The boundary layer flow of a radiative nanofluid over a non-linear stretching sheet was discussed by Anwar et al. [18]. A good number of works on nanofluids over linear non-linear sheets are found in [19-21].

In view of the above studies the present paper deals with the steady MHD boundary layer flow of heat and mass transfer in nanofluid over a stretching sheet with the effect of heat source and suction parameter. numerically by adopting the well-known implicit finite difference method known as Keller-Box method. The governing equations of the flow are solved numerically and results are discussed graphically.

II. MATHEMATICAL VISCOPLASTIC FLOW MODEL

Consider the steady MHD boundary layer flow of nanofluid over a stretching sheet in presence of chemical reaction and viscous dissipation. The governing equations of momentum, energy and species concentration equations can be written in usual notation as

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial x}=0 \quad (1)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\upsilon\frac{\partial^2 u}{\partial y^2}+g\beta\left(T-T_\infty\right)+g\beta^*\left(C-C_\infty\right)-\frac{\sigma B^2}{\rho_f}u \quad (2)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\frac{\nu}{c_p}\left(\frac{\partial u}{\partial y}\right)^2+\frac{(\rho c)_p}{(\rho c)_f}\left[D_B\frac{\partial N}{\partial y}\frac{\partial T}{\partial y}+\frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^2\right] \quad (3)$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_B\frac{\partial^2 C}{\partial y^2}+\frac{D_T}{T_\infty}\frac{\partial^2 T}{\partial y^2}-K_0\left(C-C_\infty\right) \quad (4)$$

The boundary conditions are given by

$$u=U_w, v=v_w, T=T_w, C=C_w \; as\, \eta\to 0$$
$$u\to\infty, u_y\to\infty, T\to T_\infty, C\to C_\infty, \eta\to\infty \quad (6)$$

Where v_0 is a constant with $v_0<0$ for mass suction and $v_0>0$ for mass injection.
Now we introduce the similarity transformations:

$$\psi=\sqrt{\frac{\nu a}{(1-ct)}}xf(\eta),\; T=T_\infty+\frac{bx}{(1-ct)^2}\theta(\eta),\; \eta=\sqrt{\frac{a}{\nu(1-ct)}}y,\; C=C_\infty+\frac{bx}{(1-ct)^2}\phi(\eta) \quad (7)$$

Where u, v are velocity components, T *and* C are respectively, the temperature and concentration of chemical species in the fluid, ν is the kinematic viscosity, k_0 is the non-Newtonian visco-elastic parameter, ε is the permeability coefficient of porous medium, g is the acceleration due to gravity, β is the volumetric coefficient of thermal expansion, β^* is the volumetric concentration coefficient, ρ is the fluid density, σ is the fluid electrical conductivity, k is the thermal conductivity, C_p is the specific heat at constant pressure, D is the mass diffusivity and k_1 is the chemical reaction parameter.

$$f'''+ff''-2f'^2+Gr\;\theta+Gc\;\phi-Mf'=0 \quad (8)$$

$$\theta''+\Pr\left(f\theta'-f'\theta+Nb\theta'\phi'+Nt\theta'^2+Ecf''^2\right)=0 \quad (9)$$

$$\phi'' + Le\left(f\phi' - f'\phi \right) + \frac{Nt}{Nb}\theta'' - Le \cdot \gamma \cdot \mathrm{Re}_x \cdot \phi = 0 \quad (10)$$

The corresponding boundary conditions are

$$\begin{aligned} &\eta = 0,\ f = s,\ f' = 1, \theta = 1, \phi = 1 \\ &\eta \to \infty,\ f' = 0,\ f'' = 0, \theta = 0, \phi = 0 \end{aligned} \quad (11)$$

$S > 0(v_0 < 0)$ corresponds to mass suction and $S < 0(v_0 > 0)$ corresponds to mass injection.

The quantities of physical interest for this problem are the local skin friction coefficient C_f, the local Nusselt number Nu_x, and the local Sherwood number Sh_x, which are respectively, defined as

Sherwood number Sh and they are defined as:

$$\tau_w = \frac{\tau^*}{\mu b x \sqrt{b/\nu}} = -f'(0) \qquad where\ \tau^* = -\left(\frac{\partial u}{\partial y}\right)_{y=0},$$

$$Nu = -\frac{h}{T_w - T_\infty} T_y = \theta'(0) \quad (12)$$

$$Sh = -\frac{h}{C_w - C_\infty} C_y = \phi'(0)$$

III. COMPUTATIONAL FINITE DIFFERENCE SOLUTIONS

In Equations (8)- (10) are nonlinear, it is impossible to get the analytical solutions. Consequently, the equations using the boundary conditions (11) are solved numerically by means of a finite-difference scheme known as the Keller-Box method. This technique, despite recent developments in other numerical methods, remains a powerful and very accurate approach for parabolic boundary layer flows. It is unconditionally stable and achieves exceptional accuracy [22]. It has been used recently in Non-Newtonian fluid flow dynamics by Amanulla et al. [23-25]. In this study a uniform grid of size $\Delta\eta = 0.001$ is taken and the solutions are obtained with an error of tolerance in all cases, which gives four decimal places accurate for most of the prescribed quantities as shown in the table. One of the factors modifying the accuracy of the method is the adapted the initial guesses.

IV. RESULTS AND DISCUSSION

In this study, the effect of viscous dissipation and chemical reaction on the MHD boundary layer flow of a nanofluid over an exponentially stretching permeable sheet is considered. The transformed nonlinear ordinary differential equations (8)–(10) with boundary conditions (11) are solved numerically using Keller box method. Dimensionless velocity, temperature and concentration profiles as well as the local skin friction, the Nusselt number and Sherwood number were analyzed for different emerging flow parameters involved in the problem. The numerical results were discussed for the various values of the parameters graphically and in the tabular form.

Figures 4, 5 and 6 illustrate the effect of suction/bowling parameter S on the velocity, temperature and concentration profiles in the stretching sheet boundary layer flow. It shows that the increasing values of S the velocity, temperature and concentration decreases and hence the thickness of boundary layer decreases. Excellent flow control is therefore achieved in the nanofluid sheet regime with suction. Temperature and concentration are also found to be strongly decreased with the presence of suction (Figs. 5 and 6). Suction achieves a strong suppression of nano-particle species diffusion and also regulates the diffusion of thermal energy in the boundary layer. This shows that suction has significant effects on the constitution of engineered nanofluids and shows that suction is an excellent mechanism for achieving flow control, cooling and nano-particle distribution in nanofluid fabrication.

Figs. 7 depict the effect of Prandtl number Pr with velocity, temperature and concentration. For increasing values of Pr results reduction in velocity and temperature due to the fact increasing values of Pr amounts to lesser thermal conductivity whereas the concentration increases with the increasing values of Pr.

Figure 8 shows the effect of the Brownian motion parameter *(Nb) and* thermophoresis parameter *(Nt)* on the temperature profile. Increasing the values of Nb&Nt accelerate the temperature profile throughout the regime. Geometrically smaller nano-particles possess higher Nb values which assist in thermal diffusion in the boundary layer via enhanced thermal conduction. Conversely larger nano-particles exhibit lower *Nb*values and this suppresses thermal conduction. The regime is heated and better distribution of nano-particles is achieved with increasing thermophoresis parameter Nt.

Figures 9 display the Brownian motion parameter *(Nb)* on the concentration profile, as Brownian motion parameter increases the nanoparticle volume boundary layer thickness decreases. Figure 10 shows the influence of thermophoresis parameter *(Nt)* on the concentration distribution. The concentration boundary layer thickness increases with the increasing of *Nt*.

Figure 11 reveals that the influence of Eckert number (Ec) on temperature profile. The wall temperature of the sheet increases as the values of Ec increase. Moreover, when the values of Ec increase, the thermal boundary layer thickness increases. This is due to the fact that the heat transfer rate at the surface decrease as Ec increases as shown in Table 1.

Figs. 12 present the profiles of velocity, temperature and concentration for values of Lewis number *(Le)*. With the increasing values of *Le* the temperature increases but the velocity and concentration decreases. This is due to the fact that as the molecular diffusion increases the concentration decreases.

Fig. 13 shows that the increasing values of chemical reaction parameter γ the concentration decreases. This is due to the fact that chemical reaction in this system results in consumption of the chemical and hence results in decrease of concentration profile. The most important effect is that the first order chemical reaction has a tendency to diminish the overshoot in the profiles of the solute concentration in the solutal boundary layer.

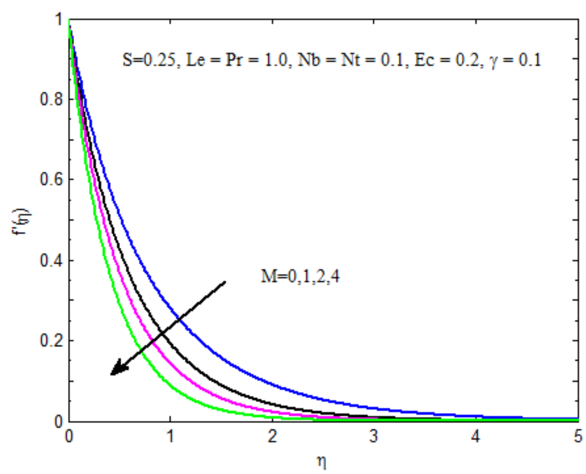

Figure 1: Effect Magnetic parameter (M) on Velocity Profile

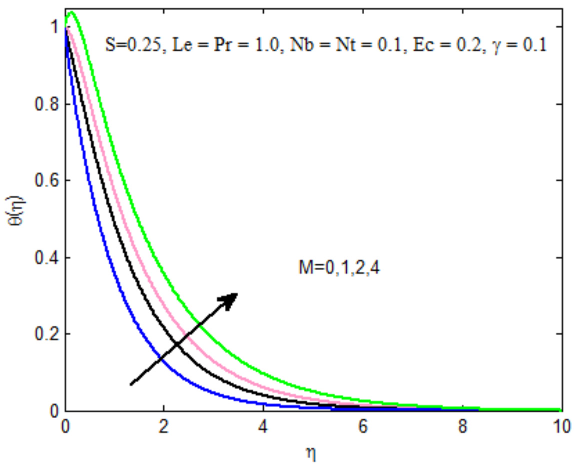

Figure 2: Effect Magnetic parameter (M) on temperature Profile

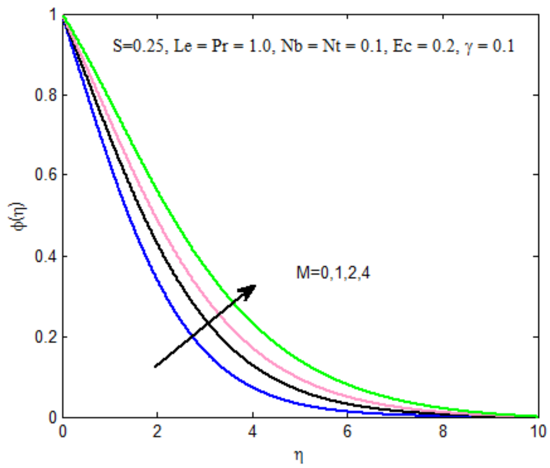

Figure 3: Effect Magnetic parameter (M) on Concentration Profile

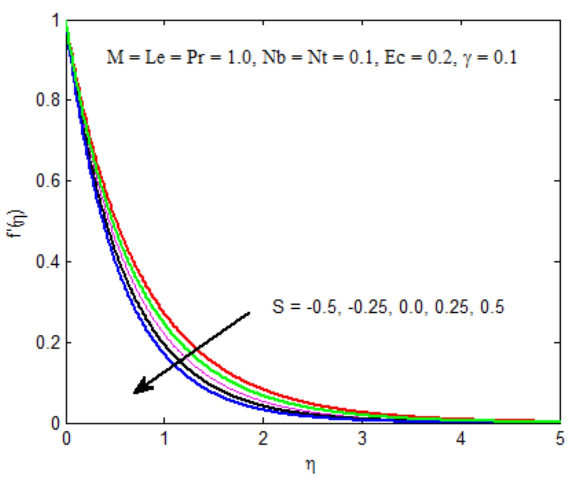

Figure 4: Effect of Suction parameter (S) on Velocity Profile

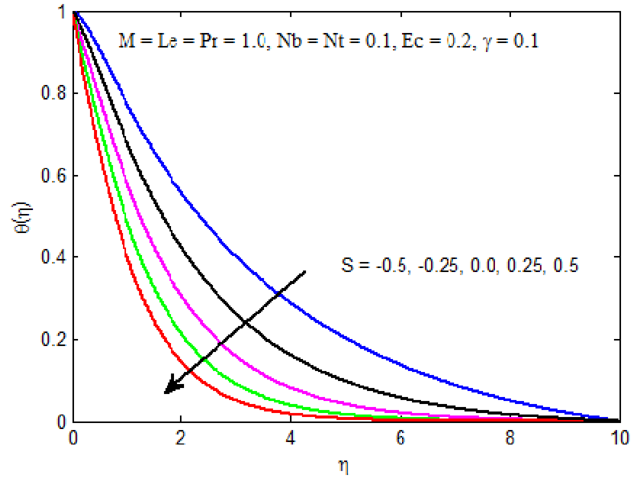

Figure 5: Effect of Suction parameter (S) on temperature Profile

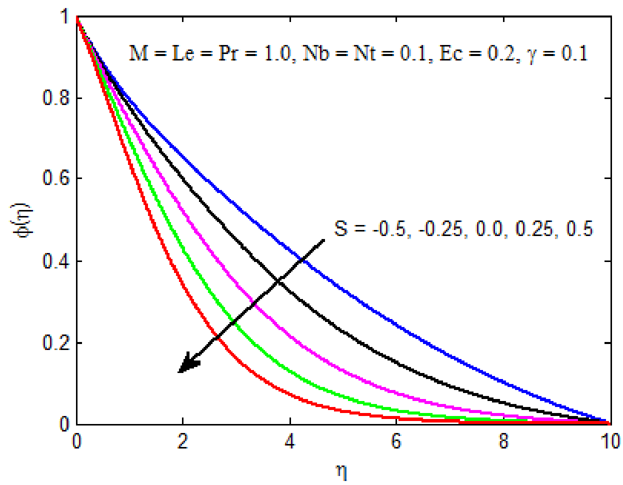

Figure 6: Effect of Suction parameter (S) on Concentration Profile

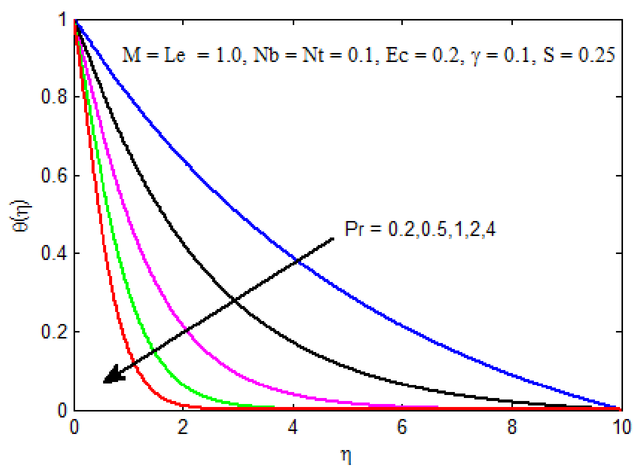

Figure 7: Effect of Prandtl number (Pr) on temperature profile

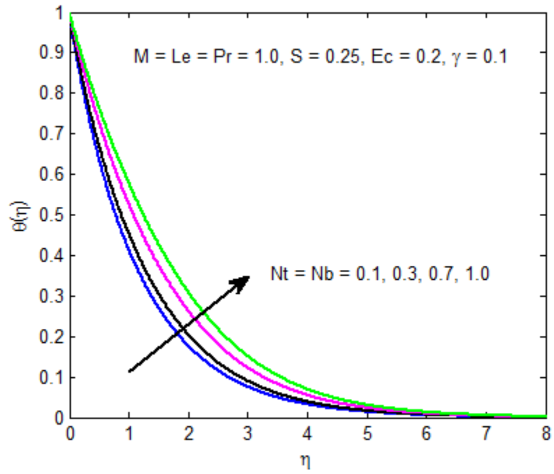

Figure 8: Effect of Nb&Nt on temperature profile

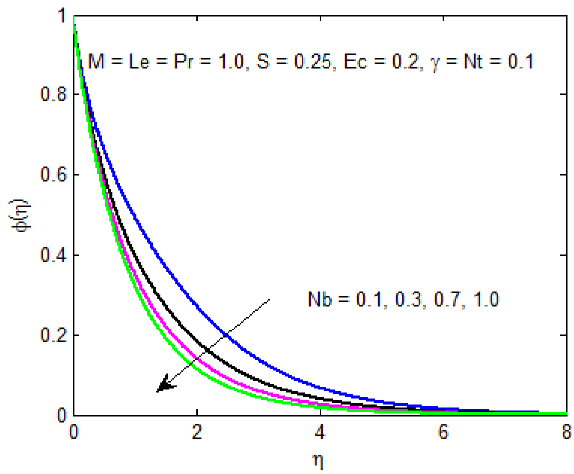

Figure 9: Effect of Brownian motion parameter (Nb) on concentration profile

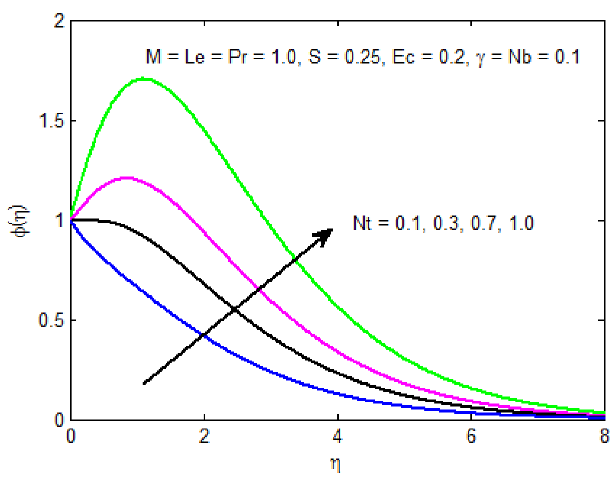

Figure 10: Effect of Thermophoresis parameter (Nt) on concentration profile

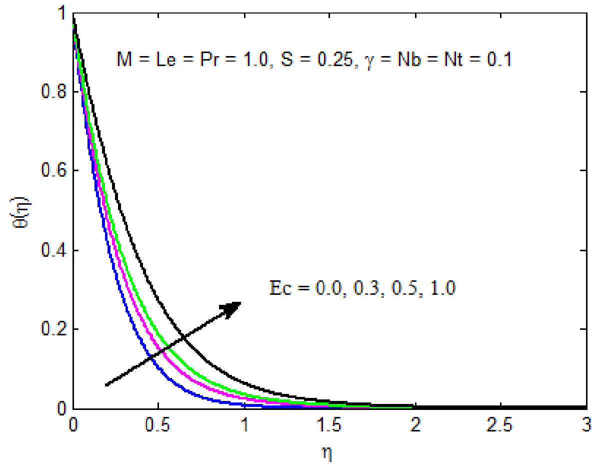

Figure 11: Effect of Eckert number (Ec) temperature profile

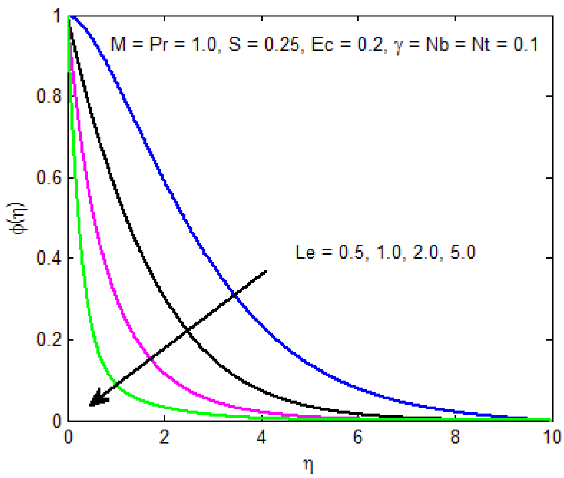

Figure 12: Effect of Lewis number (Le) on concentration profile.

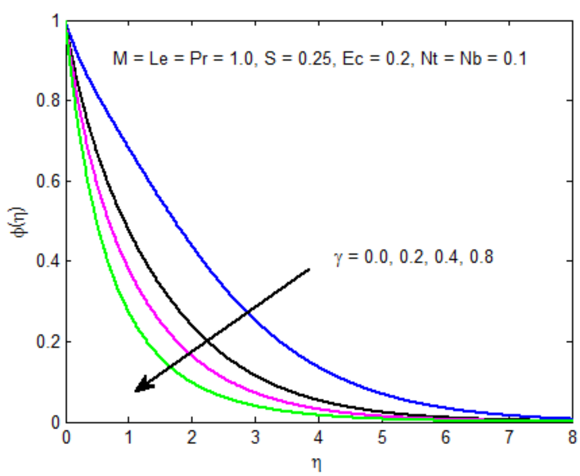

Figure 13: Effect of Chemical reaction parameter (γ) on concentration profile

In the table 1, Effects of Magnetic parameter, Eckert number and Chemical reaction parameter when $Pr = 1$, $Le = 1$, $Nb = Nt = 0.1$ and $S = 0.25$ on skin-friction coefficient, Nusselt and Sherwood numbers are presented.

Table 1: Effect of M, Ec and γ on skin-friction coefficient, Rate of heat and mass transfer coefficients when Pr = 1.0, Nb=Nt=0.1, Le = 1.0, S = 0.25.

M	Ec	γ	$-f''(0)$	$-\theta'(0)$	$-\varphi'(0)$
0.1	0.2	0.1	1.4439	0.9757	3.0527
0.5	0.2	0.1	1.5903	0.9239	3.0421
0.1	0.5	0.1	1.4439	0.9672	3.0588
0.1	0.2	0.5	1.4439	0.9707	3.8203

V. CONCLUSION

The steady MHD boundary layer flow of an incompressible, nanofluid over a stretching sheet in the presence of buoyancy effects with chemical reaction and suction/injection is analyzed. The following conclusions are drawn. It is observed that the velocity decreases with suction, magnetic field parameter, Prandtl number and Schmidt number while an opposite trend is noted with thermal and solutal buoyancy parameters, thermal radiation and heat source parameter. From the present study the important findings are listed below.

a. The Magnetic field reduces the velocity profile, whereas temperature and concentration profiles are enhanced with increasing of Magnetic parameter (M).

b. As suction parameter S increases, the velocity, temperature and concentration profiles are decreases.

c. The temperature profile is increases with the increase of Brownian motion parameter (Nb) and Thermophoresis parameter (Nt).

d. The concentration profile decreases with the increase of Brownian motion parameter (Nb), whereas reverse trend is observed with the increase of Thermophoresis parameter (Nt).

e. Temperature profile decreases with the increasing values of Prandtl number (Pr) and it is increased with the increasing of Eckert number (Ec)

f. Concentration profile is reduced by enhancing both chemical reaction parameter (γ) and Lewis number (Le).

g. The skin friction increases with the increasing of Magnetic parameter.

h. The local Nusselt number decreases with the increasing of Magnetic parameter, chemical reaction parameter and Eckert number.

i. The local Sherwood number decreases with the increasing of Magnetic parameter, whereas it is increase with the increasing of Eckert number and chemical reaction parameter

REFERENCES

1. *B.C. Sakiadis, Boundary layer behavior on continuous solid surfaces, Am. Inst. Chem. Eng. J. 7(1961)26-28.*
2. *L.J. Crane, Flow past a stretching plate, Z. Angew. Math. Phys. 21(1970) 645-647.*
3. *F.K. Tsou, E.M. Sparrow, R.J. Goldstein, Flow and heat transfer in the boundary layer on a continuous moving surface, Int. J. Heat Mass Transfer 10 (1967) 219-223.*
4. *L.E. Erickson, L.T. Fan, V.G. Fox, Heat and mass transfer on a moving continuous moving surface, Ind. Eng. Chem. Fund. 5 (1966) 19-25.*
5. *P.S. Gupta, A.S. Gupta, Heat and mass transfer on a stretching sheet with suction or blowing, Can. J. Chem. Eng. 55 (1977) 744-746.*
6. *L.J. Crane, Journal of Applied Mathematics and Physics,**21(**4), pp. 645–647, 1970.*
7. *K. Jafar, R. Nazar, A. Ishak, I. Pop. Can. J. Chem. Eng. **9999(**111) (2011)*
8. *K.V. Prasad, D. Pal, V. Umesh, N.S. Prasanna Rao, Commun. Nonlinear Sci. Numer. Simul. **15**(2) 331-334.(2010)*
9. *G. Singh and P.R. Sharma. Journal of Applied Fluid Mechanics, **2(**1), pp. 13-21, (2009).*
10. *S. Choi., Enhancing thermal conductivity of fluids with nanoparticles, ASMEPublications-Fed, 231(1995) 99-106.*
11. *S. Kakaç and A. Pramuanjaroenkij, Review of convective heat transfer enhancement with nanofluids, International Journal of Heat and Mass Transfer, 52(2009) 3187- 3196.*
12. *V. Trisaksri, and S. Wongwises, Critical review of heat transfer characteristics ofnanofluids, Renewable and Sustainable Energy Reviews, 11(2007) 512-523.*
13. *Haoran Li, Li Wang, Yurong He, Yanwei Hu, Jiaqi Zhu and Baocheng Jiang,Experimental investigation of thermal conductivity and viscosity of ethylene glycol basedZnO nanofluids, Applied Thermal Engineering 88(2015) 363-368.*

14. *W.A. Khan and I. Pop, Boundary-layer flow of a nanofluid past a stretching sheet,International Journal of Heat and Mass Transfer, 53(2010) 2477-2483.*
15. *R. Abdul Hamid, R. Nazar and I. Pop, Non-alignment stagnation-point flow of ananofluid past a permeable stretching/shrinking sheet: Buongiorno's model. ScientificReports 5(2015) 14640, 1-11.*
16. *K. Zaimi, A. Ishak and I. Pop, Boundary layer flow and heat transfer over a nonlinearlypermeable stretching/ shrinking sheet in a nanofluid, Scientific Reports 4, 4404 (2014) 1-8;*
17. *M. Ferdows, Md. Shakhaoath Khan, Md. Mahmud Alam and A. A. Afify, MHDboundary layer flow and heat transfer characteristics of a nanofluid over a stretching sheet,Acta Universitatis Sapientiae, Mathematica, 9(2017) 140-161.*
18. *M.I. Anwar., S. Sharidan., I. Khan., and M. Z. Salleh, Magnetohydrodynamic andradiation effects on stagnation-point flow of nanofluid towards a nonlinear stretching sheet,Indian Journal of Chemical Technology, 21(2014) 199-204.*
19. *Dianchen Lu, M. Ramzan, Noor ul Huda, Jae Dong Chung and U. Farooq, Nonlinearradiation effect on MHD Carreau nanofluid flow over a radially stretching surface with zeromass flux at the surface, Scientific Reports, 8(2018) 3709.*
20. *Kalidas Das, Nanofluid flow over a non-linear permeable stretching sheet with partialslip, Journal of the Egyptian Mathematical Society, 23(2015) 451-456.*
21. *Ahmed A. Khidir and Precious Sibanda, Nanofluid flow over a nonlinear stretchingsheet in porous media with MHD and viscous dissipation effects, journal of porousmedia,17(2014) 391-403.*
22. *Keller, H.B. (1970). A new difference method for parabolic problems, J. Bramble (Editor), Numerical Methods for Partial Differential Equations, Academic Press, New York, USA.*
23. *Amanulla, C.H., N. Nagendra and M. Surya Narayana Reddy (2017). Numerical Study of Thermal and Momentum Slip Effects on MHD Williamson Nanofluid from an Isothermal Sphere, Journal of Nanofluids, 6(6), 1111–1126.*
24. *Amanulla, C.H., Nagendra, N. and Suryanarayana Reddy, M., "Numerical Simulations on Magnetohydrodynamic Non Newtonian Nanofluid Flow Over a Semi-Infinite Vertical Surface with Slip effect," Journal of Nanofluids, 2018, 7(4), 718-730.*
25. *Amanulla, C.H., Nagendra, N. and Suryanarayana Reddy, M., "Numerical Simulation of Slip Influence on the Flow of a MHD Williamson Fluid Over a Vertical Convective Surface," Nonlinear Engineering, 2018, 1-13.*